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The problem of a composite piezoelectric plate transducer

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Abstract : An attempt has been made to investigate the mechanical disturbance of a composite piezoelectric plate transducer executing vibration in the thickness mode which is taken along x-axis. The portion of the thickness $x = 0$ to $x = 1$ is excited electromechanically and at the end $x = 0$, is applied an impulsive voltage input. The problem involves of interaction of two fields, *viz.*, electrical and mechanical. The method of laplace transform has been used to find the disturbances and for small time scale ranging the nature of the disturbances is found to be linear in nature and it is of the order of 10^{-5} cm

Keywords : Piezoelectricity, plate transducer, mechanical disturbance.

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1. Introduction

The studies in the disturbances of a piezoelectric material from the stand point of mechanics of continuous media have been initiated by [1–4]. These types of problems are very much interesting due to their various practical applications in different branch of science and technology [5–7]. The problems of a composite piezoelectric slab form a very fascinating branch in the theory of piezoelectricty. Researchers [3,6,8] have investigated the disturbance in the piezoelectric slab sandwitched between different medium under variety of excitations. Such type of problems are important in view of their direct applications to practical problems in which conversions of electro-mechanical energies are involved like microphone, underwater signaling [7,8], *etc.*

2. The problem and the fundamental equations

We consider a piezoelectric plate transducer executing vibration in the thickness mode which is the mode generally used [8,9] in the generation of ultrasonic waves. Let the thickness direction of transducer be taken in the x-axis and let its extremities be $x = 0$ to $x = X$. The portion of the thickness $x = 0$ to $x = 1$ is excited electromechanically. To the end $x = 0$ is applied an impulsive voltage input V given by

$$V = V_0 \delta(t) \tag{1}$$

where $\delta(t)$ is the Dirac delta function and V_0 is a constant. Obviously, this constitutes one of the types of a 'composite' transducer [5,6]. The object of this paper is to investigate the nature of the mechanical response owing to the voltage input given by eq. (1).

The mechanical displacement ξ in the x-direction satisfies the equation of motion

$$\rho\, \partial^2 \xi / \partial t^2 = \partial T_1 / \partial x \tag{2}$$

where ρ is the density of the material and T_1 is the stress.

The constitutive relations [1–3] are

$$T_1 = c_{11} S_1 + e_{11} E_1 \tag{3}$$

$$P_1 = e_{11} S_1 + k_{11} E_1 \tag{4}$$

where S_1, E_1 and P_1 are respectively the components of strain, electric intensity and polarization and c_{11}, e_{11} and k_{11} are the elastic, piezoelectric and susceptibility coefficients respectively.

From eqs. (2)–(4) we get

$$\rho \partial^2 \xi / \partial t^2 = \left(c_{11} - e_{11}^2 / k_{11}\right) \partial^2 \xi / \partial x^2 + e_{11} / k_{11} \partial P_1 / \partial x \tag{5}$$

where $S_1 = \partial \xi / \partial x$. In accordance with our assumption we consider the polarization gradient of the form

$$\partial P_1 / \partial x = P_0 \sin \omega t, \quad \omega > 0 \tag{6}$$

where P_0 is a constant. Eq. (5) becomes

$$\partial^2 \xi / \partial x^2 + e_{11} / (c_{11} k_{11} - e_{11}^2) P_0 \sin \omega t = 1/v^2 \; \partial^2 \xi / \partial t^2 \tag{7}$$

where $v^2 = 1/\rho (c_{11} - e_{11}^2 / k_{11})$ represent the velocity of propagation in the transducer. Writing $E_1 = \partial V / \partial x$ we get from eqs. (5) and (6)

$$\partial V / \partial x = 1/k_{11} (P_0 x \sin \omega t - e_{11}\, \partial \xi / \partial x)$$

Solving eq. (7) after taking Laplace transform, we get

$$\bar{\xi} = A \exp^{px/v} + B \exp^{-px/v} + P_0 e_{11} / \rho k_{11} \omega / (p^2 + \omega^2) p^2 \tag{8}$$

where A, B are constants and p is the Laplace transform parameter.

To ascertain the constants A and B we must enumerate the boundary conditions of the problem. The most general type [7] of the problem can be thought of as consisting of a transducer of impedance Z_T situated between the two systems of impedance Z_1 and Z_2. The conditions of continuity of the displacements at the extremeties $x = 0$ and $x = X$ as well as at $x = 1$ when formulated give rise to

$$\text{i)} \quad \text{at } x=0, \quad (\bar{\xi}_1)_0 = (\bar{\xi})_0;$$

$$\text{ii)} \quad \text{at } x=1, \quad (\bar{\xi}_1')_1 = (\bar{\xi})_1;$$

$$\text{iii)} \quad \text{at } x=X, \quad (\bar{\xi}_2)_x = (\bar{\xi}')_x; \tag{9}$$

where the suffixes 1 and 2 denote the entities of the materials at $x = 0$, $x = X$ respectively and we write

$$\bar{\xi}_1 = A_1 \exp^{px/v_1} + B_1 \exp^{-px/v_1}$$

$$\bar{\xi}_2 = A_2 \exp^{px/v_2} + B_2 \exp^{-px/v_2} \tag{10}$$

$$\bar{\xi}' = A' \exp^{px/v} + B' \exp^{-px/v} + P_0 e_{11} \omega / \rho k_{11} (p^2 + \omega^2) p^2$$

To simplify the calculations we consider the transducers to be rigidly backed [10] at the extremety $x = X$ so that $A_2 = B_2 = A_1 = 0$ [6]. We get

$$\begin{aligned}(\bar{\xi})_{x=1} = {} & \bar{V} Z_k / D\big[\left(\exp^{p1/v} - \theta_3 / \theta_2 p^2 + \theta_3\right) \cdot \left\{Z \exp^{p(x-21)/v}\right. \\ & \left. - \exp^{-px/v} + 1c_2 \theta_4 (p + a_3)/Z_k\right\} + \left(\exp^{-p1/v} - \theta_3 / \theta_2 p^2 + \theta_3\right) \\ & \times \left\{\exp^{px/v} - Z \exp^{p(x-21)/v} - \eta 1 c_2 \theta_4 (p + a_3)/Z_k\right\}\big] \\ & + \theta_1 / \rho V / \theta_2 p^2 + \theta_3 \end{aligned} \tag{11}$$

where D is the material constant and

$$Z = Z_c - Z_c' / Z_c + Z_c', \; Z_k = c_1 (Z_c + Z_c'), \; \eta = Z_1 + Z_c / Z_1 - Z_c,$$

$$Z_c = \rho v S = YZ / v\, (c_{11} - e_{11}^2 / k_{11}'), \; c_1 = Z_1 / Z_1 - Z_c$$

$$\theta_4 = 1 - \exp^{2p(x-1)/v} / \exp^{p(x-1)/v}, \; a_3 = 2Z_c' / 1c_2 \theta_4,$$

$$\theta_1 = e_{11} / k_{11}, \; \theta_2 = X^2 / 2k_{11}, \; \theta_3 = e_{11}^2 / \rho k_{11}$$

and

$$C_2 = v(Z_c' - Z_c) \rho k_{11} / (c_{11} k_{11} - e_{11}^2).$$

Now Laplace inversion of the expression eq. (11) is too much cumbersome, so, to get an approximate value we have taken the recourse of asymptotic expansion for small and large values of time [9] to get an idea about the nature of displacement at the point $x = 1$.

2.1. Calculation of displacement for small values of time

Substituting the value $\bar{V}$ from eq. (1) we obtain approximate displacement at the point $x = 1$ and is given by

$$(\xi)_x . \theta_5 / V_0 = 2(\eta - 1)c_2 \theta_3 / \theta_2 (X - vt/v^2) x^2 - \big[2(\eta - 1)c_2 X \theta_3 / \theta_2$$
$$(X - vt/v^2) - \{2c_1 Z_c - (\eta + 1) Z_c'\} 2/v \big] x + 2(\eta - 1) Z_c' t$$
$$- \{2c_1 Z_c + (\eta - 1) Z_c'\} 2x/v + 3t/2 \tag{12}$$

where $\theta_5 = C_3 (Z_c + Z_c')$.

2.2. Calculation of displacement for large values of time

After a lot of calculation we obtain the displacement for large values of time as

$$(\xi)_{x=1} \doteq V_0 H(t) / \rho e_{11} D \tag{13}$$

where $H(t)$ is the Heaviside unit function. Since D is the material constant contains Z, X, ρ and e_{11}, the mechanical displacement for the particular case can be obtained easily by assigning suitable values to Z, X and V_0.

3. Discussions

For the purpose of numerical calculations, we take the following standard numerical values of the material constants for quartz [2,11],

$\rho = 2.65$ gm/cm^3, $k_{11} = -1.2 \times 10^6$, $e_{11} = 0.513 \times 10^5$, $c_{11} = 86.74 \times 10^{10}$ dyn/cm^2, $v = (86.74 \times 10^{10}/2.65)^{1/2}$ cm/sec.

To facilitate numerical computations, the values of Z_1, Z_c, Z_c', X, P_0 and ω have been chosen suitably [7,9] as $Z_1 = 2$, $Z_c = 1$, $Z_c' = .5$, $X = 10$ cm, $P_0 = 1$, $\omega = 1.57$ and $V_0 = 300$ v.

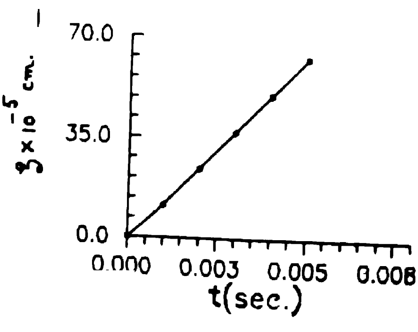

Figure 1. Variation of mechanical disturbance with time.

The numerical values of the mechanical displacement corresponding to small values of t for $x = 2$ cm has been shown in Fig. 1. It is found that the nature of the disturbances is linear in nature and it is of the order of 10^{-5} cm.

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